Influence Line Diagram

For Indeterminate Structures

Solved Example

Example

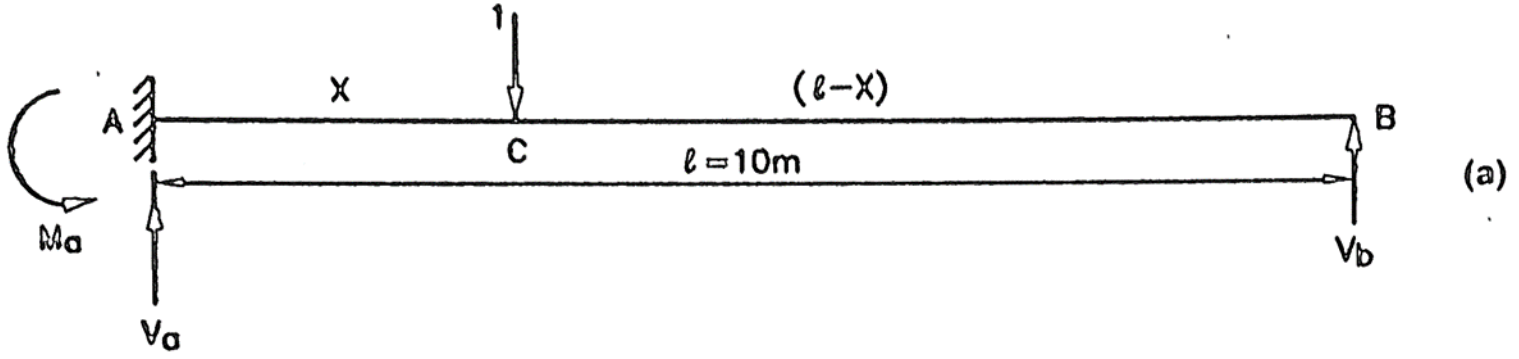
Draw the influence line diagrams for a propped cantilever beam shown in figure for;

i. Reaction at A (Va)

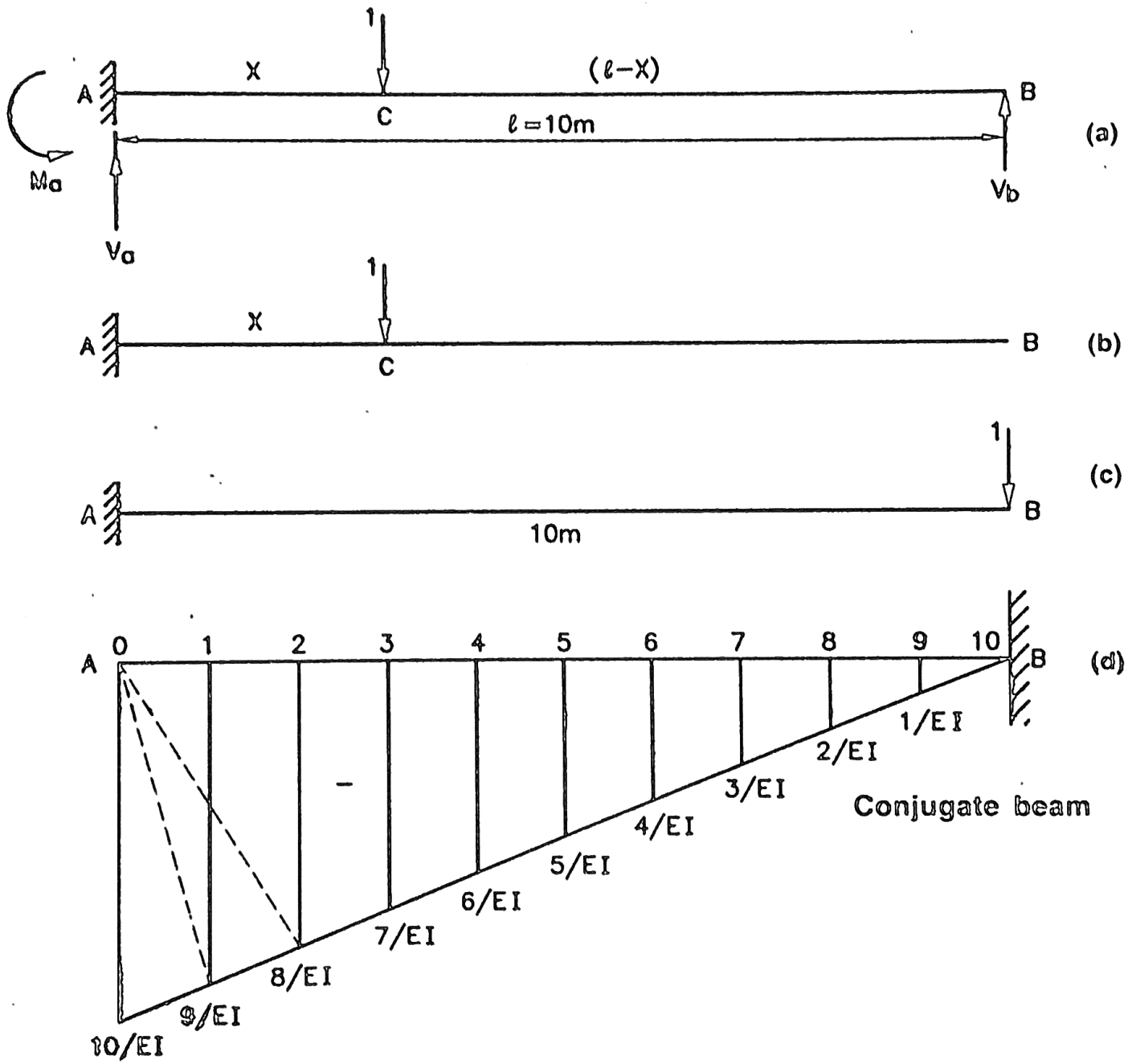
ii. Reaction at B (Vb)

iii. Moment at A (Ma)

Calculate the ordinates at 1m intervals.



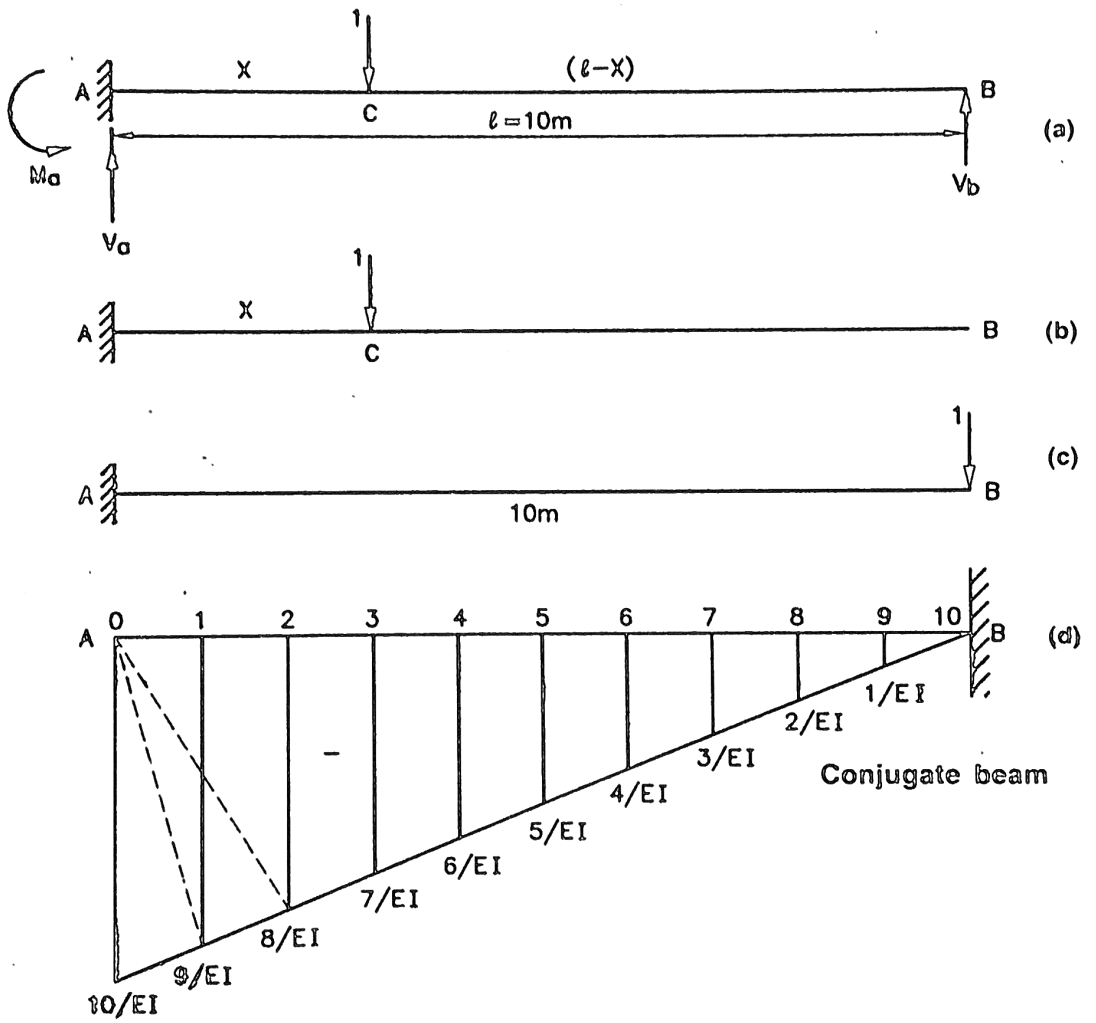
Solution:



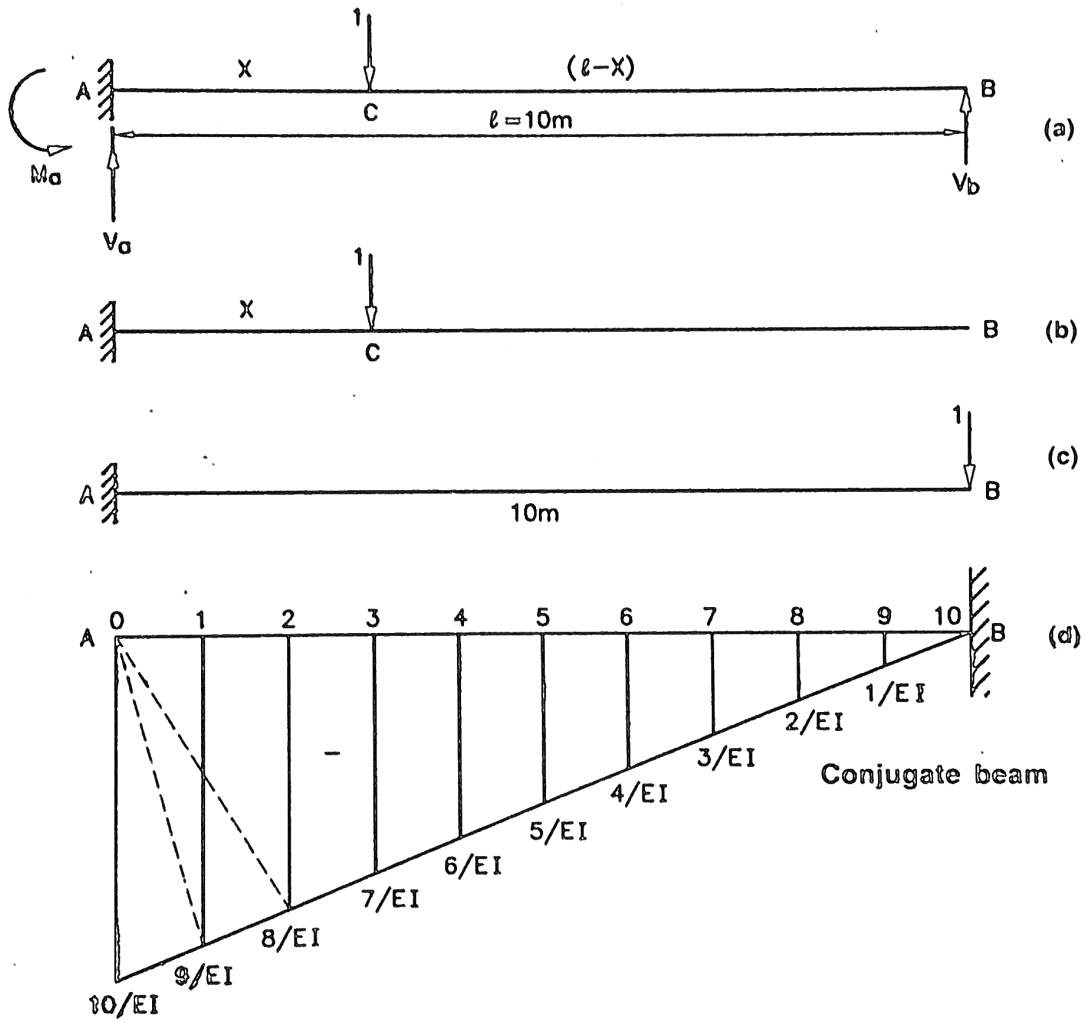
$$Vb = \frac{\delta xb}{\delta bb}$$

We know that, for a conjugate beam, deflection at any point = B.M. at that point divided by EI

- $\delta_0 = 0$
- $\delta_1 = \left(\frac{1}{2} * 1 * 10\right) * \frac{2}{3} * 1 + \left(\frac{1}{2} * 1 * 9\right) * \frac{1}{3} * 1 = 4.83$
- $\delta_2 = \left(\frac{1}{2} * 2 * 10\right) * \frac{2}{3} * 2 + \left(\frac{1}{2} * 2 * 8\right) * \frac{1}{3} * 2 = 18.66$
- $\delta_3 = \left(\frac{1}{2} * 3 * 10\right) * \frac{2}{3} * 3 + \left(\frac{1}{2} * 3 * 7\right) * \frac{1}{3} * 3 = 40.5$
- $\delta_4 = \left(\frac{1}{2} * 4 * 10\right) * \frac{2}{3} * 4 + \left(\frac{1}{2} * 4 * 6\right) * \frac{1}{3} * 4 = 69.33$
- $\delta_5 = \left(\frac{1}{2} * 5 * 10\right) * \frac{2}{3} * 5 + \left(\frac{1}{2} * 5 * 5\right) * \frac{1}{3} * 5 = 104.16$



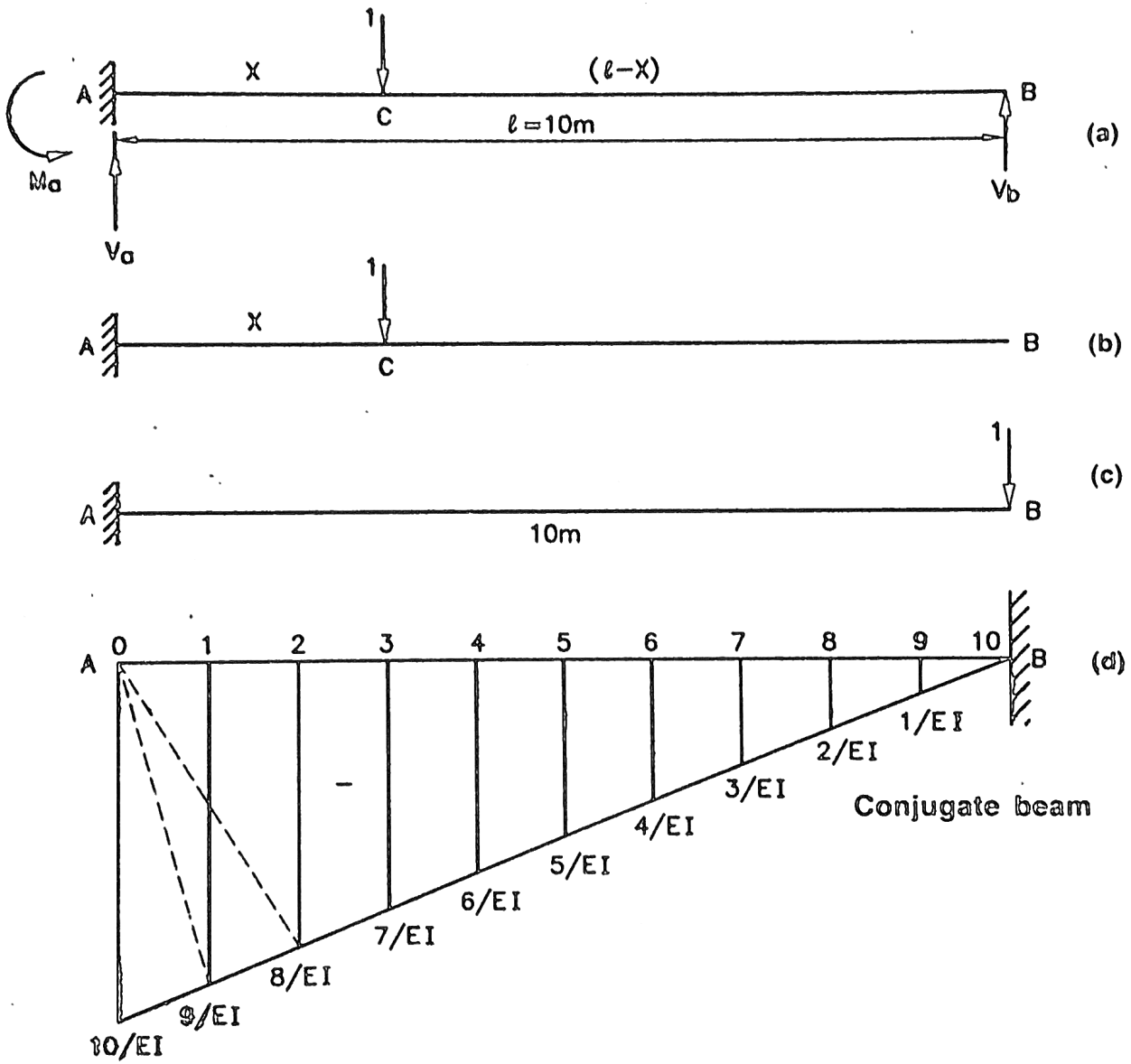
- $\delta_6 = \left(\frac{1}{2} * 10 * 6\right) * \frac{2}{3} * 6 + \left(\frac{1}{2} * 4 * 6\right) * \frac{1}{3} * 6 = 144.0$
- $\delta_7 = \left(\frac{1}{2} * 10 * 7\right) * \frac{2}{3} * 7 + \left(\frac{1}{2} * 3 * 7\right) * \frac{1}{3} * 7 = 187.83$
- $\delta_8 = \left(\frac{1}{2} * 10 * 8\right) * \frac{2}{3} * 8 + \left(\frac{1}{2} * 2 * 8\right) * \frac{1}{3} * 8 = 234.66$
- $\delta_9 = \left(\frac{1}{2} * 10 * 9\right) * \frac{2}{3} * 9 + \left(\frac{1}{2} * 1 * 9\right) * \frac{1}{3} * 9 = 283.5$
- $\delta_{10} = \left(\frac{1}{2} * 10 * 10\right) * \frac{2}{3} * 10 = 333.33$

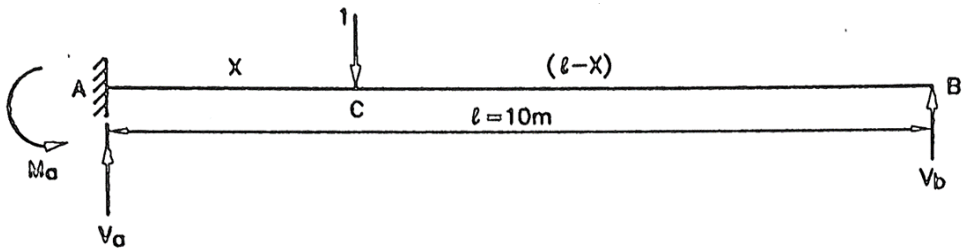


$$Vb = \frac{\delta xb}{\delta bb}, \text{ here}$$

$$\delta bb = \delta_{10} = 333.33$$

Distance x	Vb
At x = 0	Vb= 0/333.33 = 0
At x = 1	Vb= 4.83/333.33 = 0.0145
At x = 2	Vb= 18.66/333.33 = 0.056
At x = 3	Vb= 40.5/333.33 = 0.122
At x = 4	Vb= 69.33/333.33 = 0.208
At x = 5	Vb= 104.16/333.33 = 0.313
At x = 6	Vb= 144.0/333.33 = 0.432
At x = 7	Vb= 187.83/333.33 = 0.564
At x = 8	Vb= 234.66/333.33 = 0.704
At x = 9	Vb= 283.5/333.33 = 0.851
At x = 10	Vb= 333.33/333.33 = 1





$$Va = 1 - Vb$$

Distance x	Va
At x = 0	Va= 1 – 0 = 1
At x = 1	Va= 1 – 0.0145 = 0.986
At x = 2	Va= 1 – 0.056 = 0.944
At x = 3	Va= 1 – 0.122 = 0.878
At x = 4	Va= 1 – 0.208 = 0.792
At x = 5	Va= 1 – 0.313 = 0.687
At x = 6	Va= 1 – 0.432 = 0.568
At x = 7	Va= 1 – 0.564 = 0.436
At x = 8	Va= 1 – 0.704 = 0.296
At x = 9	Va= 1 – 0.851 = 0.149
At x = 10	Va= 1 – 1 = 0

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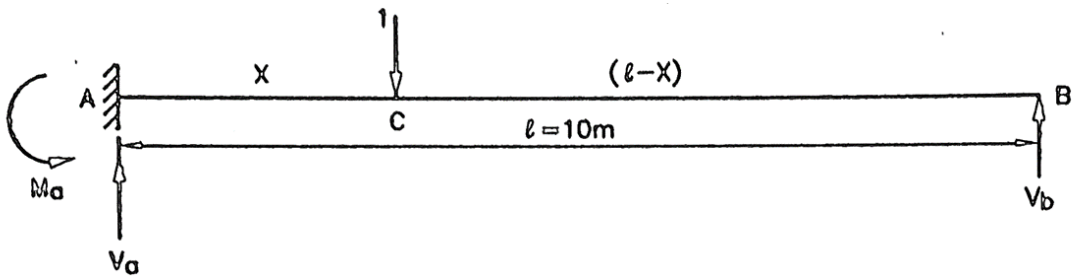

$$Ma = (Vb * 10) - (1 * x)$$

Distance x	$Ma = (Vb * 10) - (1 * x)$
At x = 0	$Ma = 0$
At x = 1	$Ma = (0.0145 * 10) - (1 * 1) = -0.855$
At x = 2	$Ma = (0.056 * 10) - (1 * 2) = -1.44$
At x = 3	$Ma = (0.121 * 10) - (1 * 3) = -1.79$
At x = 4	$Ma = (0.208 * 10) - (1 * 4) = -1.92$
At x = 5	$Ma = (0.313 * 10) - (1 * 5) = -1.87$
At x = 6	$Ma = (0.432 * 10) - (1 * 6) = -1.68$
At x = 7	$Ma = (0.564 * 10) - (1 * 7) = -1.36$
At x = 8	$Ma = (0.704 * 10) - (1 * 8) = -0.96$
At x = 9	$Ma = (0.851 * 10) - (1 * 9) = -0.49$
At x = 10	$Ma = (1 * 10) - (1 * 10) = 0$

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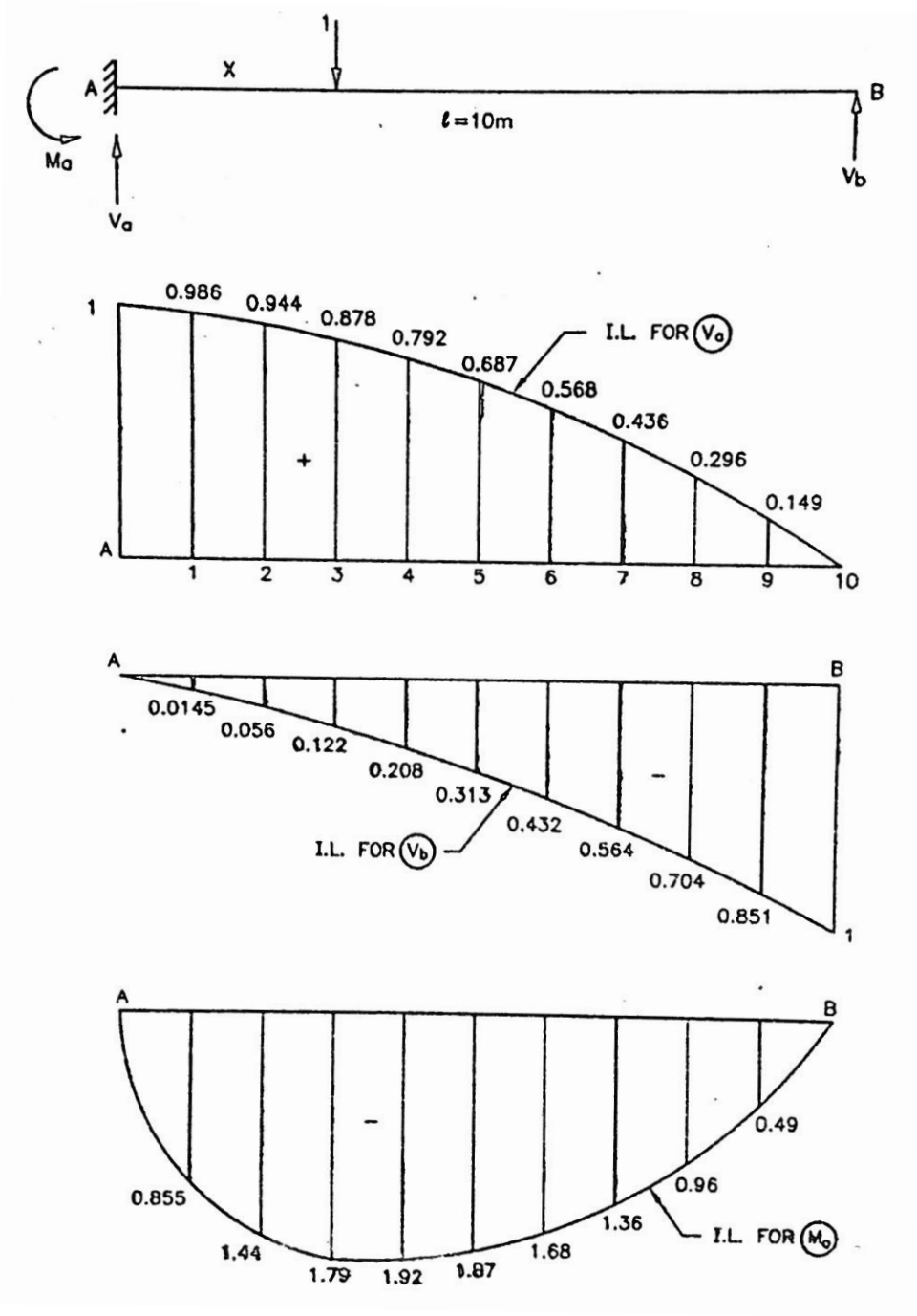

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