Influence Line Diagram

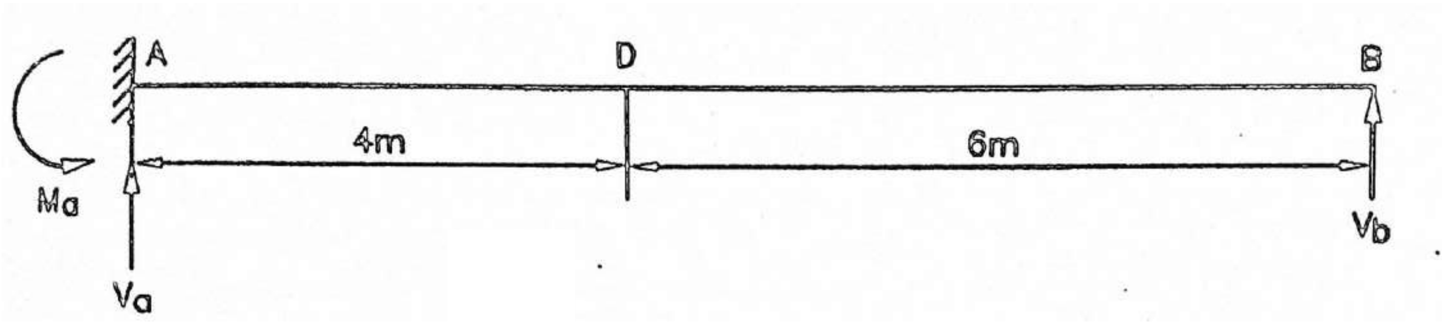
For Indeterminate Structures

I.L.D. for SHEAR FORCE & BENDING MOMENT

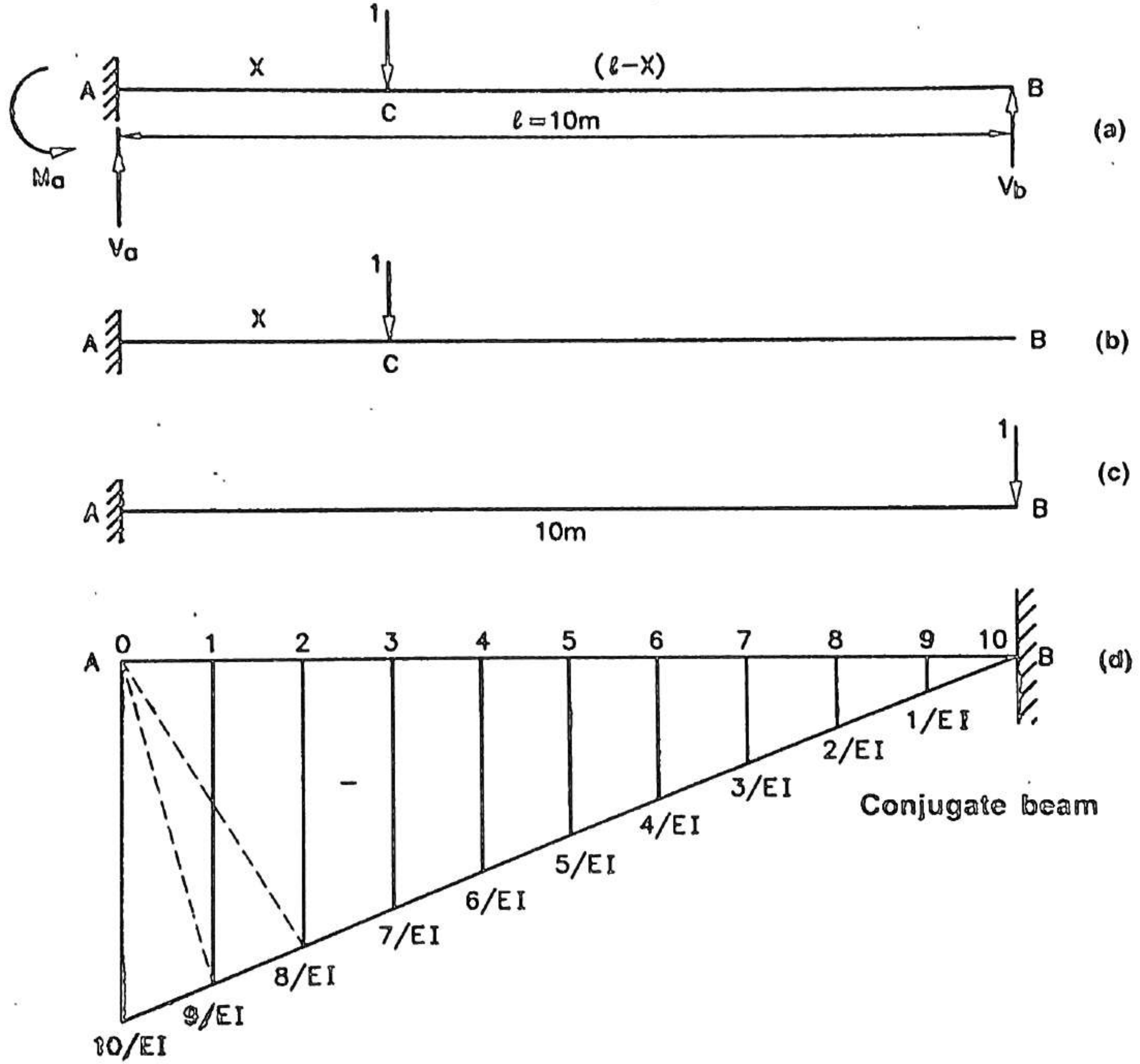
Solved Example

Example

A propped cantilever beam is having 10m span. Draw ILD for S.F. & B.M. at section 4m from the fixed end.



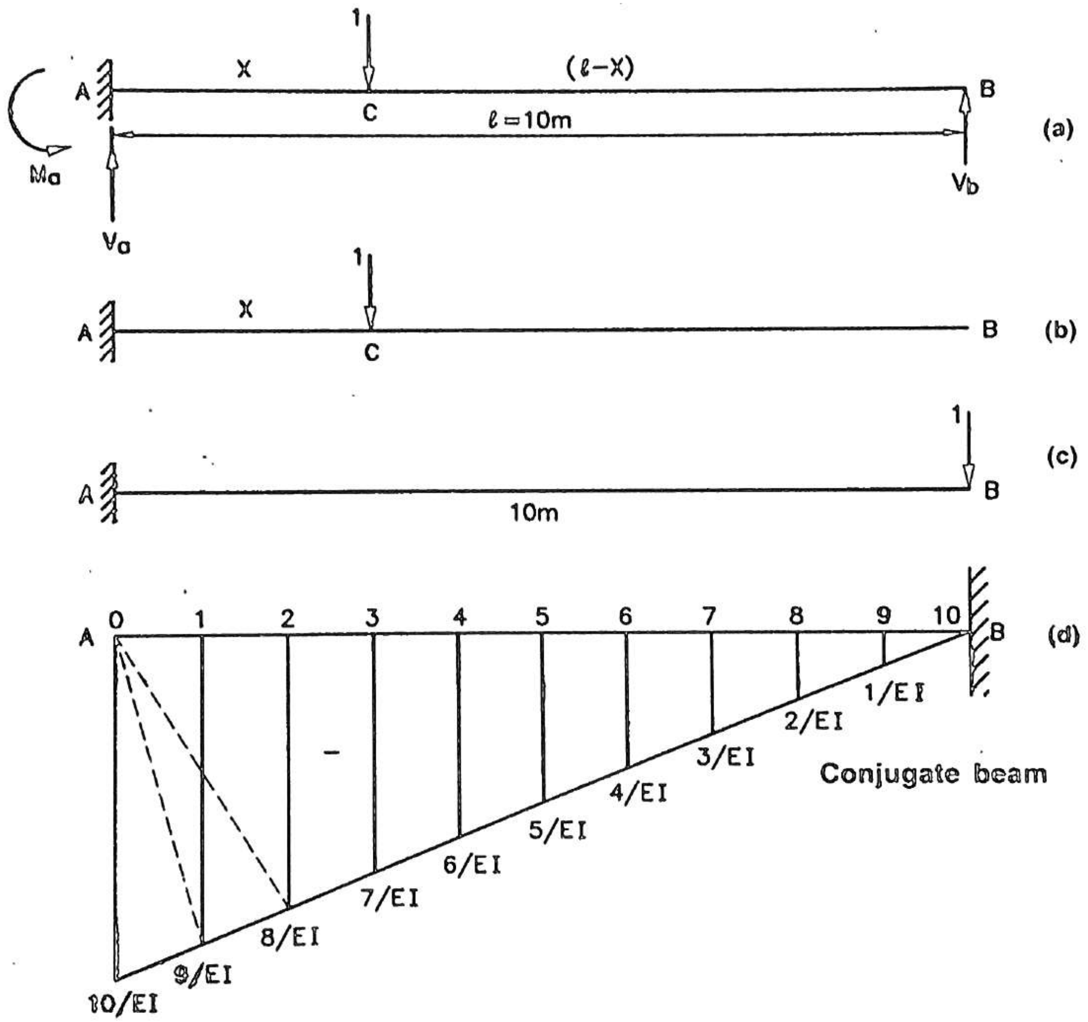
Solution:



$$Vb = \frac{\delta xb}{\delta bb}$$

We know that, for a conjugate beam, deflection at any point = B.M. at that point divided by EI

- $\delta_0 = 0$
- $\delta_1 = \left(\frac{1}{2} * 1 * 10\right) * \frac{2}{3} * 1 + \left(\frac{1}{2} * 1 * 9\right) * \frac{1}{3} * 1 = 4.83$
- $\delta_2 = \left(\frac{1}{2} * 2 * 10\right) * \frac{2}{3} * 2 + \left(\frac{1}{2} * 2 * 8\right) * \frac{1}{3} * 2 = 18.66$
- $\delta_3 = \left(\frac{1}{2} * 3 * 10\right) * \frac{2}{3} * 3 + \left(\frac{1}{2} * 3 * 7\right) * \frac{1}{3} * 3 = 40.5$
- $\delta_4 = \left(\frac{1}{2} * 4 * 10\right) * \frac{2}{3} * 4 + \left(\frac{1}{2} * 4 * 6\right) * \frac{1}{3} * 4 = 69.33$
- $\delta_5 = \left(\frac{1}{2} * 5 * 10\right) * \frac{2}{3} * 5 + \left(\frac{1}{2} * 5 * 5\right) * \frac{1}{3} * 5 = 104.16$



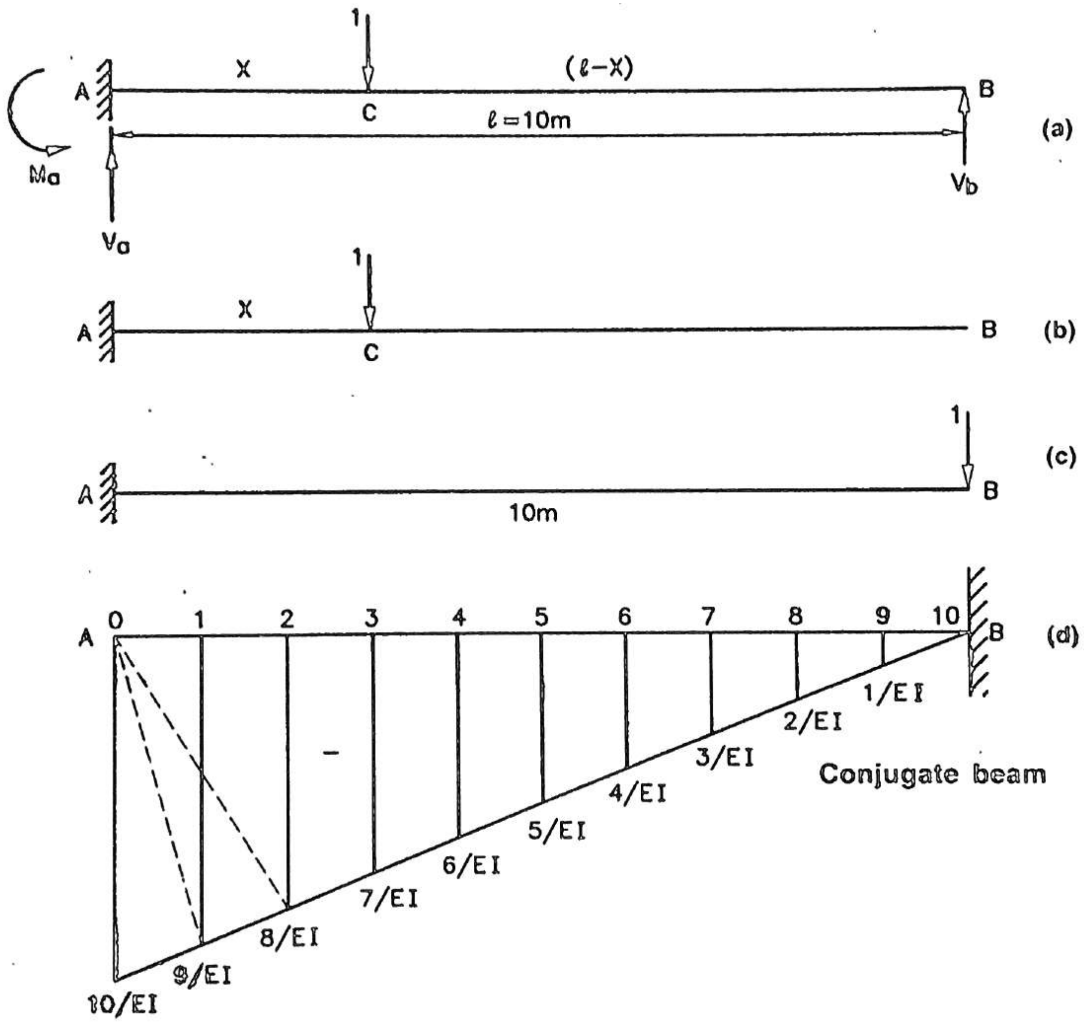
- $\delta_6 = \left(\frac{1}{2} * 10 * 6\right) * \frac{2}{3} * 6 + \left(\frac{1}{2} * 4 * 6\right) * \frac{1}{3} * 6 = 144.0$

- $\delta_7 = \left(\frac{1}{2} * 10 * 7\right) * \frac{2}{3} * 7 + \left(\frac{1}{2} * 3 * 7\right) * \frac{1}{3} * 7 = 187.83$

- $\delta_8 = \left(\frac{1}{2} * 10 * 8\right) * \frac{2}{3} * 8 + \left(\frac{1}{2} * 2 * 8\right) * \frac{1}{3} * 8 = 234.66$

- $\delta_9 = \left(\frac{1}{2} * 10 * 9\right) * \frac{2}{3} * 9 + \left(\frac{1}{2} * 1 * 9\right) * \frac{1}{3} * 9 = 283.5$

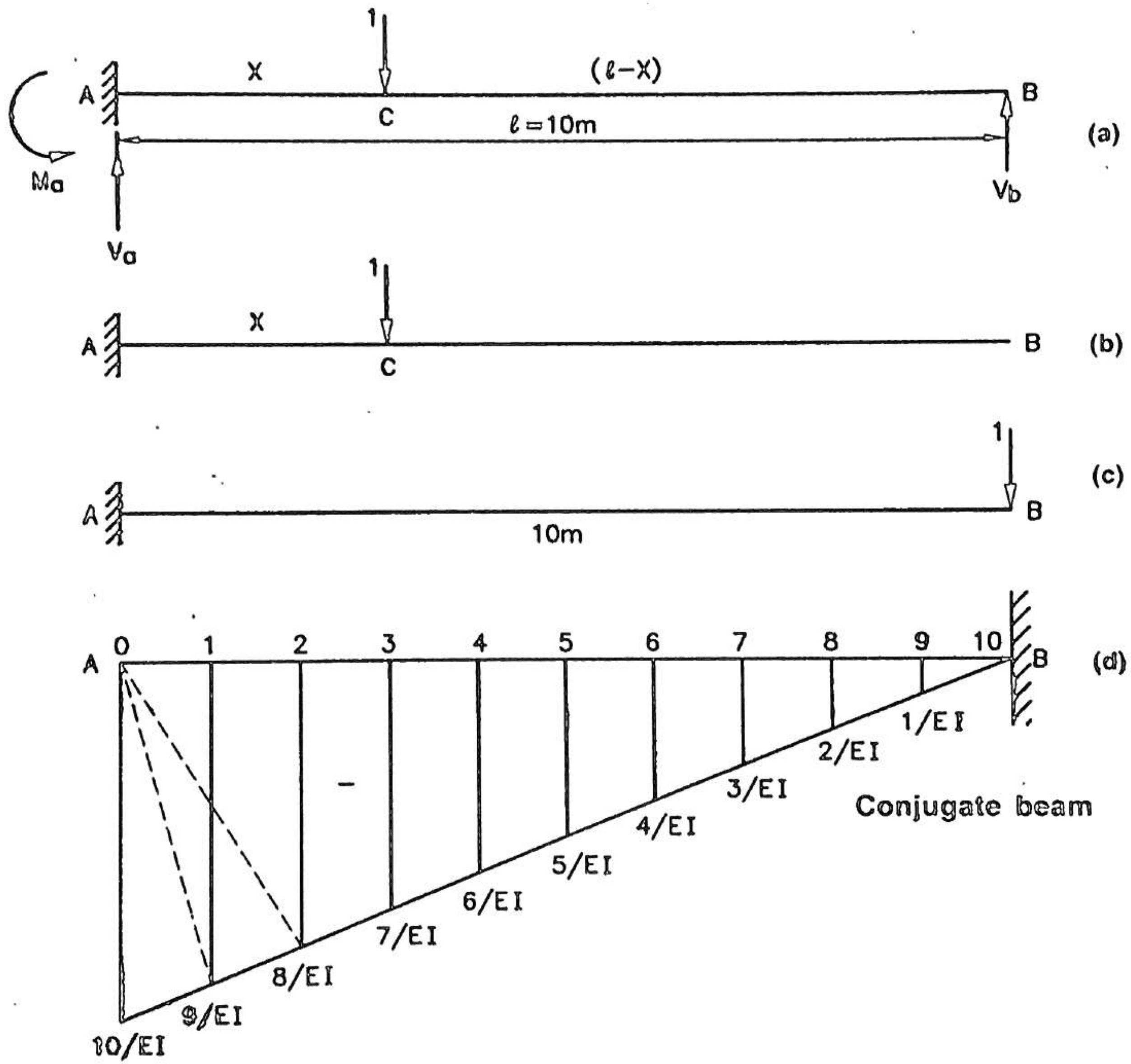
- $\delta_{10} = \left(\frac{1}{2} * 10 * 10\right) * \frac{2}{3} * 10 = 333.33$



$$Vb = \frac{\delta xb}{\delta bb}\text{, here}$$

$$\delta bb = \delta_{10} = 333.33$$

Distance x	Vb
At x = 0	Vb= 0/333.33 = 0
At x = 1	Vb= 4.83/333.33 = 0.0145
At x = 2	Vb= 18.66/333.33 = 0.056
At x = 3	Vb= 40.5/333.33 = 0.122
At x = 4	Vb= 69.33/333.33 = 0.208
At x = 5	Vb= 104.16/333.33 = 0.313
At x = 6	Vb= 144.0/333.33 = 0.432
At x = 7	Vb= 187.83/333.33 = 0.564
At x = 8	Vb= 234.66/333.33 = 0.704
At x = 9	Vb= 283.5/333.33 = 0.851
At x = 10	Vb= 333.33/333.33 = 1



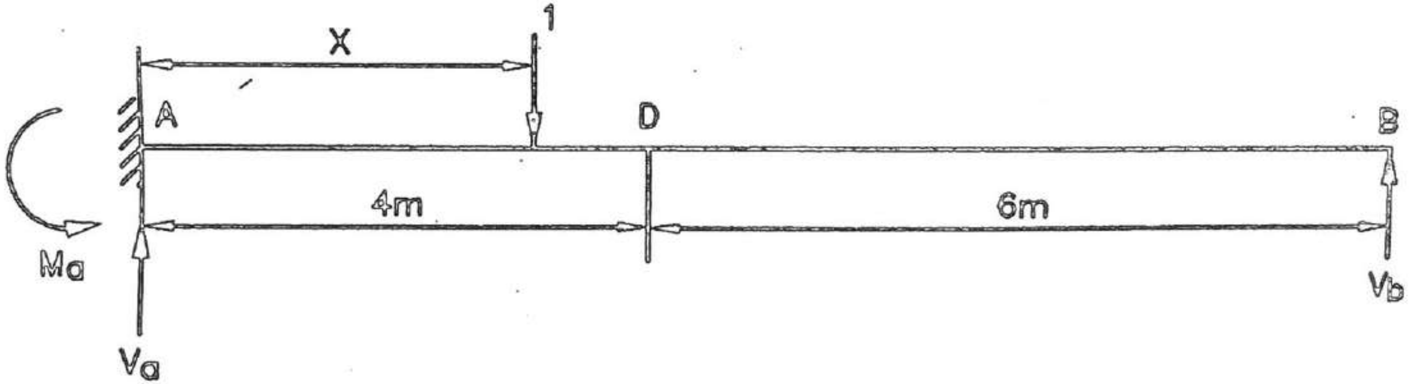
I.L.D. for Shear Force at D

When unit load is in AD:

S.F. at **D** $(\boldsymbol{Vd}) = \boldsymbol{Vb}$

(sum of vertical forces on RHS of D is 0)

Distance x	$Vd = Vb$
0	0
1	0.0145
2	0.056
3	0.122
4	0.208

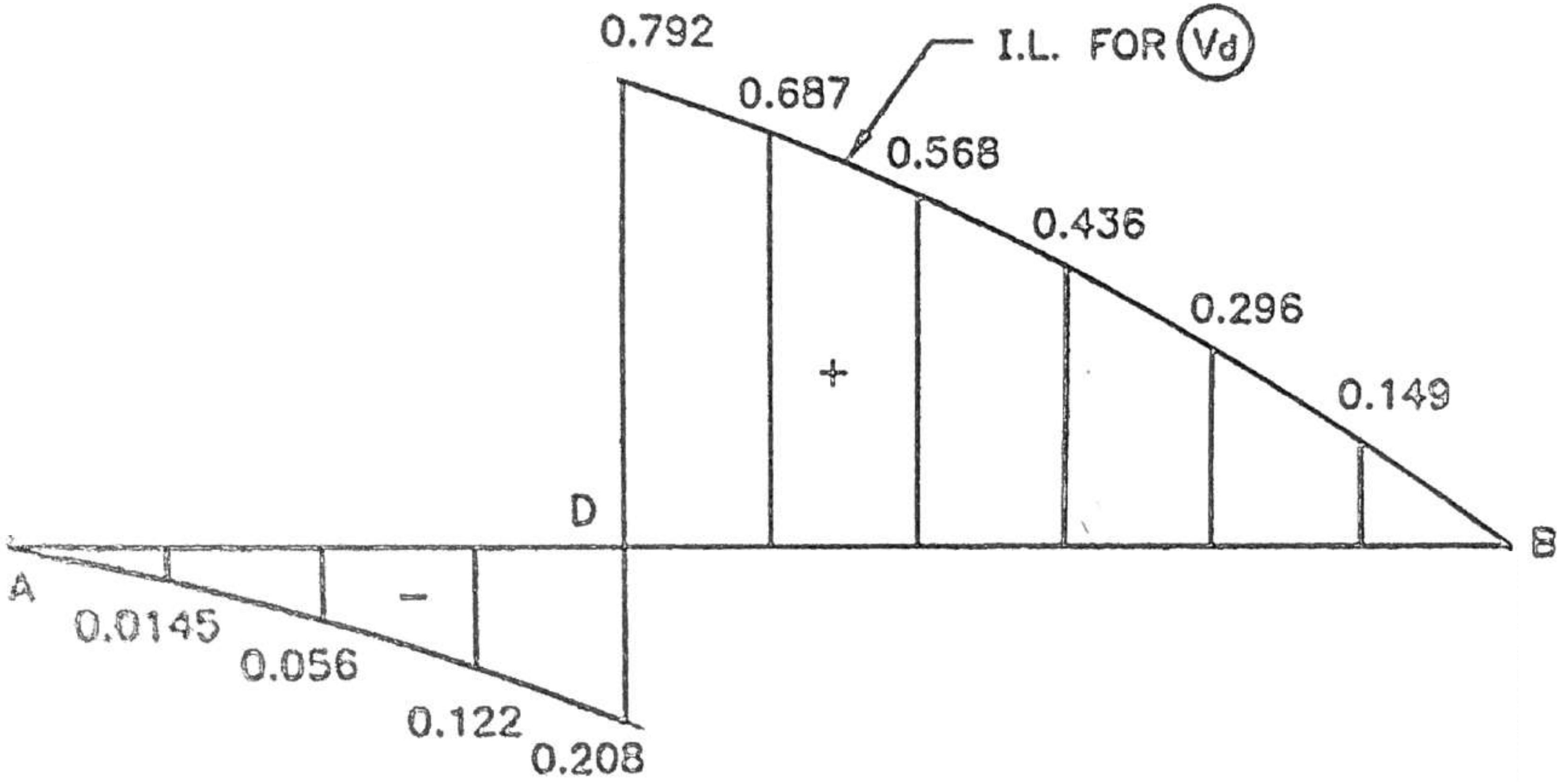


When unit load is in DB:

S.F. at $D\ (Vd) = (1 - Vb)$ OR $Vd = Va$

$\because (1 - Vb) = Va$

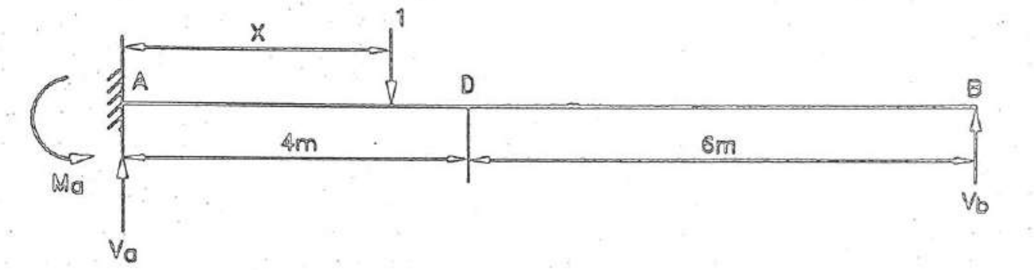
Distance x	$Vd = 1 - Vb$
4	$\boldsymbol{Vd}$ = 1 – 0.208 = 0.792
5	$\boldsymbol{Vd}$ = 1 – 0.313 = 0.687
6	$\boldsymbol{Vd}$ = 1 – 0.432 = 0.568
7	$\boldsymbol{Vd}$ = 1 – 0.564 = 0.436
8	$\boldsymbol{Vd}$ = 1 – 0.704 = 0.296
9	$\boldsymbol{Vd}$ = 1 – 0.851 = 0.149
10	$\boldsymbol{Vd}$ = 1 – 1 = 0



I.L.D. for Shear Force at D

I.L.D. for Bending Moment at D

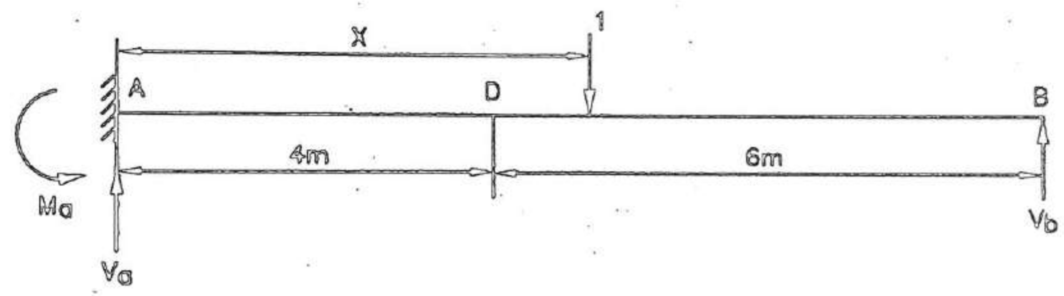
When unit load is in AD:



$$Md = Vb * 6$$

Distance x	$Md = Vb * 6$
0	Md = 0 x 6 = 0
1	Md = 0.0145 x 6 = 0.087
2	Md = 0.056 x 6 = 0.336
3	Md = 0.122 x 6 = 0.732
4	Md = 0.208 x 6 = 1.248

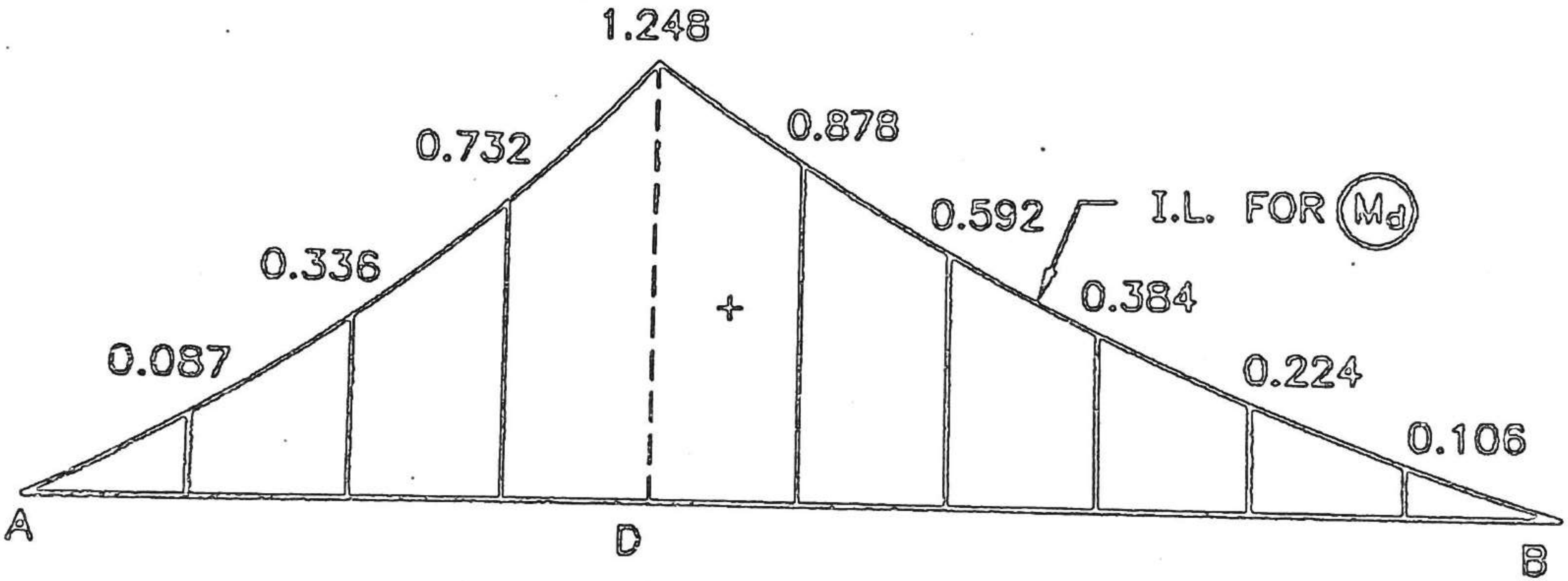
When unit load is in DB:



Taking moment at right side of D

$$Md = Vb * 6 - 1(x - 4)$$

Distance x	$Vd = 1 - Vb$
4	Md = 0.208 x 6 – 1 (4 - 4) = 1.248
5	Md = 0.313 x 6 – 1 (5 - 4) = 0.878
6	Md = 0.432 x 6 – 1 (6 - 4) = 0.592
7	Md = 0.564 x 6 – 1 (7 - 4) = 0.384
8	Md = 0.704 x 6 – 1 (8 - 4) = 0.224
9	Md = 0.851 x 6 – 1 (9 - 4) = 0.106
10	Md = 1 x 6 – 1 (10 - 4) = 0



I.L.D. for Bending Moment at D

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